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NATURAL CONVECTION IN A VOLUMETRICALLY HEATED FLUID LAYER AT HIGH RAYLEIGH NUMBERS

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Abstract—A phenomenological model of eddy heat transport in natural convection with volumetric energy sources at high Rayleigh numbers is developed in this study. The model is applied to the problem of thermal convection in a horizontal heated fluid layer with an adiabatic lower boundary and an isothermal upper wall. A correlation for mean Nusselt numbers is obtained for steady heat transfer. Distributions of the mean turbulent temperature, the eddy heat flux, and production of thermal variance in the heated fluid are presented. The mechanism of turbulent thermal convection at high Rayleigh numbers is discussed. Comparison is made with existing experiments and found to be good.

NOMENCLATURE

C_p, specific heat;
g, acceleration due to gravity;
Ⓗ, dimensionless temperature at any η, $2k(T-T_1)/qL^2$;
Ⓗ$_0$, dimensionless lower wall temperature, $2k(T_0-T_1)/qL^2$;
k, thermal conductivity;
l^*, local length scale;
L, layer thickness;
Nu, Nusselt number, $qL^2/2k(T_0-T_1)$;
q, volumetric energy sources;
Q^*, local heat flux at any z;
Ra^*, local Rayleigh number, $g\beta\Delta T^* l^{*3}/\alpha\nu$;
Ra_I, internal Rayleigh number, $g\beta qL^5/2k\alpha\nu$;
T, mean temperature at any z;
T', fluctuating temperature;
T_1, upper surface temperature;
T_0, lower surface temperature;
ΔT^*, characteristic local temperature, $T-T_1$;
v, vertical fluctuating velocity in z direction;
$\overline{vT'}$, eddy heat flux, $-\varepsilon_H \dfrac{dT}{dz}$;
$\overline{w\text{Ⓗ}'}$, dimensionless eddy heat flux, $\overline{vT'}/\alpha(T_0-T_1)/L$;
z, vertical coordinate, $0 \leqslant z \leqslant L$;

Greek symbols

β, isobaric coefficient of thermal expansion;
ε_H, eddy diffusivity for heat;
δ, conduction sublayer thickness;
η, dimensionless distance from the lower wall, z/L;
ρ, density;
ρ_0, density of fluid at the lower surface;
ν, kinematic viscosity;
α, thermal diffusivity;
Pr, Prandtl number, ν/α.

Subscripts

1, upper surface;
0, lower surface.

Superscripts

$'$, fluctuating quantity;
$*$, local variable.

INTRODUCTION

THE STUDIES of volumetrically heated fluid layers have recently been of great interest to both analysts and experimentalists in the area of nuclear reactor design and safety. In particular, the assessment of postaccident heat removal in fast reactors requires fundamental knowledge of natural convection heat transfer with volumetric heat release at high Rayleigh numbers (usually of the order 1×10^6 times the critical value of linear stability theory or higher). The thermal convection flow is thus in the fully turbulent heat-transfer regime. To determine the rate and mechanism of heat removal from boundary surfaces of the heated fluids, which are related to the extent of melting attack of surrounding materials, eddy heat transport within the fluids must be investigated. The purpose of this study is to develop an analytical method to facilitate the evaluation of the turbulent heat-transfer characteristics as well as the average boundary heat fluxes of a volumetrically heated horizontal fluid layer. Results of this study can be useful to other technological areas including environmental science, geophysics, and astrophysics [1–3].

Many studies have been done in recent years in the area of natural convection with volumetric energy sources. For an internally heated fluid layer with an adiabatic lower boundary and an isothermal upper wall, work has been performed by Tritton and Zarraga [4], Roberts [5], Fiedler and Wille [6], Schwiderski and Schwąb [7], and recently by Kulacki and Nagle [8] and Kulacki and Emara [9]. For the case with equal upper and lower surface temperatures, heat-transfer studies have been carried out by Kulacki and Goldstein [10], Catton and Suo-Anttila [11], and Jahn and Reineke [12]. Recently, theoretical work on horizontal fluid layers with combined internal and external Rayleigh number effects has been reported by Suo-Anttila and Catton [13], Baker *et al.* [14], and Cheung and Baker [15]. Most of these studies, however, are either restricted to thermal convection at low Rayleigh numbers (compared to the corresponding situation in a hypothetical core meltdown event) or centered on mean transport and the overall nature of flow.

To study high-Rayleigh-number heat transfer and to explore the mechanism of turbulent thermal convection, a phenomenological model of eddy heat transport is developed in the present study. The model considers the eddy heat flux as a function of a local Rayleigh number of the flow. The local Rayleigh number is defined in terms of a characteristic local length scale and a characteristic local buoyancy difference. Average heat-transfer coefficients as well as some key turbulent energy transport features are obtained for a horizontal fluid layer with an adiabatic lower boundary and an isothermal upper wall. Comparison is made, where possible, with existing measurements.

EDDY HEAT TRANSPORT MODEL

Thermal convection in a volumetrically heated fluid layer at high Rayleigh numbers is characterized by the intensive turbulent mixing in the region away from the wall. The internal energy generated within the fluid in such a region is usually removed effectively toward the wall by eddy heat transport. In general, the local heat flux, Q^*, across the fluid layer is equal to the sum of the rate of molecular and eddy heat transfers:

$$Q^* = \rho C_p \overline{vT'} - k\frac{\mathrm{d}T}{\mathrm{d}z}. \tag{1}$$

Following the conventional definition of the eddy diffusivity for heat, we have

$$\overline{vT'} = -\varepsilon_H \frac{\mathrm{d}T}{\mathrm{d}z}. \tag{2}$$

In terms of ε_H, the local heat flux becomes

$$Q^* = -k\left(1+\frac{\varepsilon_H}{\alpha}\right)\frac{\mathrm{d}T}{\mathrm{d}z}. \tag{3}$$

The ratio ε_H/α is a local kinematic parameter and must depend on the local heat-transfer characteristics of the fluid. On the basis of dimensional analysis, the eddy thermal diffusivity can be expressed as a function of the Prandtl number of the fluid and the local Rayleigh number of the flow. The latter is defined by

$$Ra^* = \frac{\Delta B^* l^{*3}}{\alpha\nu}, \tag{4}$$

where l^* is a characteristic local length scale and ΔB^* is a characteristic local buoyancy difference, where buoyancy is defined by

$$B^* = \left(\frac{\rho-\rho_0}{\rho_0}\right)g. \tag{5}$$

Physically, Ra^* is a measure of the local buoyancy effect on the flow. As a result, ε_H must be a monotonically increasing function of Ra^*. Using a simple power-law expression, the eddy thermal diffusivity can be written as

$$\frac{\varepsilon_H}{\alpha} = aRa^{*b}. \tag{6}$$

The effect of Prandtl number on ε_H will be discussed later. With proper choice of ΔB^* and l^*, a and b in equation (6) are universal constants.

The local buoyancy difference can be related to a characteristic local temperature using the Boussinesq approximation. By choosing the characteristic local temperature as the temperature difference between the fluid and the upper plate, i.e. $\Delta T^* = T - T_1$, the result is

$$\Delta B^* = g\beta\Delta T^*. \tag{7}$$

The characteristic length scale is chosen to be a local variable which measures the degree of freedom for a fluid to undergo convective motion. Using a power series expansion, this local variable can be expressed by

$$\frac{l^*}{L} = C_0 + C_1\eta + C_2\eta^2 + C_3\eta^3 + C_4\eta^4 + \ldots, \tag{8}$$

where $\eta = z/L$ is the dimensionless distance from the lower wall and C_i $(i = 0, 1, 2, 3, \ldots)$ are unknown coefficients to be determined. Since the fluid must come to rest at the wall and is freest to move in the center portion of the layer, we must have the following boundary conditions:

$$\begin{cases} \dfrac{l^*}{L} = 0 \quad \text{for } \eta = 0 \quad \text{or } \eta = 1 \\ \dfrac{\mathrm{d}}{\mathrm{d}\eta}\dfrac{l^*}{L} = 0 \quad \text{for } \eta = \frac{1}{2}. \end{cases} \tag{9}$$

Combination of equations (8) and (9) results in a new expression for l^*/L as

$$l^*/L = \sum Cm[\eta(1-\eta)]^m, \tag{10}$$

where, to be general enough, m is not necessary to be an integer.

In the wall region (but away from the stagnant fluid sublayer to be discussed in the following section), the mean temperature is observed to vary inversely with the square of the distance from wall [10, 16, 17]. This implies that the eddy thermal diffusivity must vary according to the third power of the distance from wall in such a region. Mathematically this can be written as

$$\begin{cases} \varepsilon_H \sim \eta^3 & \text{for } \eta \to 0 \\ \varepsilon_H \sim (1-\eta)^3 & \text{for } \eta \to 1. \end{cases} \tag{11}$$

From equations (4) and (6), the eddy thermal diffusivity can be related to the characteristic local length scale by

$$\varepsilon_H \sim l^{*3b}. \tag{12}$$

Thus, in the wall region, l^* must vary in a manner according to

$$\begin{cases} l^* \sim \eta^{1/b} & \text{for } \eta \to 0 \\ l^* \sim (1-\eta)^{1/b} & \text{for } \eta \to 1. \end{cases} \tag{13}$$

Combination of equations (10) and (13) gives

$$\frac{l^*}{L} = C[\eta(1-\eta)]^{1/b}. \tag{14}$$

Substituting equations (4), (7), and (14) into equation (6), the eddy thermal diffusivity becomes

$$\frac{\varepsilon_H}{\alpha} = a\left(\frac{g\beta\Delta T^* L^3}{\alpha\nu}\right)^b [\eta(1-\eta)]^3, \tag{15}$$

where the unknown coefficient C in equation (14) has been absorbed in a. In view of equation (6), the local Rayleigh number can be redefined as

$$Ra^* = \frac{g\beta\Delta T^* L^3}{\alpha\nu}[\eta(1-\eta)]^{3/b}. \tag{16}$$

The constants a and b are to be determined in the next section.

THE UNIVERSAL CONSTANTS

Consider a stationary process of turbulent thermal convection in a volumetrically heated fluid layer with an adiabatic lower boundary and an isothermal upper wall (Fig. 1). A simple heat balance on the fluid gives

$$\frac{d}{dz}Q^* = q, \tag{17}$$

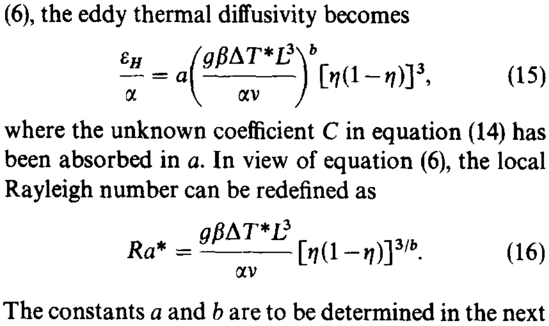

FIG. 1. Schematic of the volumetrically heated horizontal fluid layer.

where q is the rate of internal heat generation in the fluid. Substitution of equation (3) for Q^* in equation (17) yields

$$\frac{d}{dz}k\left(1+\frac{\varepsilon_H}{\alpha}\right)\frac{dT}{dz} = -q. \tag{18}$$

The boundary conditions are

$$\begin{aligned} z = 0, &\quad \frac{dT}{dz} = 0 \\ z = L, &\quad T = T_1. \end{aligned} \tag{19}$$

Integrating equation (18) once and using the boundary condition at $z = 0$, we have

$$\begin{cases} \dfrac{dT}{dz} = \dfrac{-qz}{k(1+\varepsilon_H/\alpha)} \\ z = L, \quad T = T_1. \end{cases} \tag{20}$$

Substituting equation (15) into equation (20) and normalizing the resultant equation, we obtain

$$\begin{cases} \dfrac{d\textcircled{H}}{d\eta} = \dfrac{-2\eta}{1+aRa_I^b\textcircled{H}^b[\eta(1-\eta)]^3} \\ \eta = 1, \quad \textcircled{H} = 0 \end{cases} \tag{21}$$

where

$$\textcircled{H} = \text{dimensionless temperature, } \frac{T-T_1}{qL^2/2k}$$

and

$$Ra_I = \text{internal Rayleigh number, } \frac{g\beta qL^5}{2k\alpha\nu}.$$

By definition, the mean Nusselt number at the upper surface can be written as

$$Nu = \frac{qL}{2k(T_0-T_1)/L}, \tag{22}$$

where T_0 is the lower plate temperature and qL is the surface heat flux at the upper wall. In dimensionless variables, equation (22) becomes

$$Nu = \textcircled{H}_0^{-1}, \tag{23}$$

where $\textcircled{H}_0$ is the value of $\textcircled{H}$ at the lower boundary. For given values of a and b, a unique relation between the mean Nusselt number and the internal Rayleigh number can be obtained by integration of equation (21). Alternatively, if the Nu–Ra_I relationship is known over a wide range of Rayleigh numbers, the universal constants a and b can be determined. In what follows, we introduce a conduction sublayer model with which a relationship between Nu and Ra_I can be derived.

Consider again the horizontal heated fluid layer shown in Fig. 1. In the wall region (both the lower and the upper surfaces), there is a stagnant fluid sublayer (the conduction sublayer) through which heat transfer is by conduction alone. Away from this region, the fluid is in the state of intensive turbulent mixing, especially for high Rayleigh number convection. The overall heat-transfer resistances of the layer must therefore be equal to that of the stagnant sublayer. Since the lower plate is insulated, the local heat flux near the lower boundary must be very small. Hence, the total temperature drop across the fluid layer must occur practically in the conduction sublayer near the upper wall. The thickness of the sublayer, δ, must therefore depend upon the total temperature difference of the layer as well as other local heat-transfer parameters. From dimensional considerations, we have

$$\delta \sim \left[\frac{g\beta(T_0-T_1)}{\alpha\nu}\right]^{-1/3}. \tag{24}$$

Table 1. Comparison of heat-transfer correlations for horizontal fluid layers with an adiabatic lower boundary and an isothermal upper wall

Study	Correlations	Ra_I	Mean Nusselt number at indicated values of Ra_I					
			1×10^6	1×10^7	1×10^8	1×10^9	1×10^{10}	1×10^{11}
Fiedler–Wille [6]	$Nu = 0.262Ra_I^{0.228}$	2×10^5–2.6×10^8	6.114	10.335	17.470	—	—	—
Kulacki–Nagle [8]	$Nu = 0.127Ra_I^{0.25}$	1.5×10^5–2.6×10^9	4.016	7.142	12.699	22.584	—	—
Kulacki–Emara [9]	$Nu = 0.2015Ra_I^{0.226}$	1.05×10^4–2.17×10^{12}	4.574	7.696	12.950	21.790	36.666	61.698
Present Theory	$Nu = 0.124Ra_I^{0.25}$ $Nu = 0.158Ra_I^{0.237}$	$Ra_I > 1 \times 10^6$	4.252	7.539	13.134	22.553	38.358	64.850

For steady heat conduction through a heat-generating slab, the following relation can be derived:

$$Q_1 = [2k(T_0 - T_1)q + Q_0^2]^{1/2}, \quad (25)$$

where Q_0 and Q_1 are heat fluxes at the lower and upper surfaces of the slab, respectively. Applying this result to the conduction sublayer of thickness δ, we have

$$qL = [2k(T_0 - T_1)q + q^2(L-\delta)^2]^{1/2}. \quad (26)$$

At high Rayleigh numbers, the conduction sublayer should be very thin compared to the fluid layer as a whole, i.e. $\delta/L \ll 1$. The above equation thus reduces to

$$\delta = \frac{k(T_0 - T_1)}{qL}. \quad (27)$$

Substituting the expression for δ into equation (24), we obtain, after rearrangement,

$$\frac{qL}{k(T_0 - T_1)} \sim \left(\frac{g\beta q L^5}{k\alpha\nu}\right)^{1/4}. \quad (28)$$

From the definitions of Nu and Ra_I, we have

$$Nu \sim Ra_I^{1/4}. \quad (29)$$

Thus the mean Nusselt number varies according to the quarter power of the internal Rayleigh number. Similar approach has been used by Hollands *et al.* [18] to treat thermal convection problems of the Rayleigh–Benard type.

The procedure of determination of the universal constants a and b is quite obvious. Assuming a given pair of a and b, equation (21) is integrated by the Runge–Kutta method to obtain values of Θ_0, corresponding to two different values of Ra_I (one is chosen to be ten times larger than the other). The values of Θ_0 are used to obtain the corresponding values of Nu. For a given Ra_I, *a set of* (a, b) can be determined so that the calculated Nu–Ra_I relation satisfies equation (29). The universal constants a and b are chosen such that equation (29) can be best fitted over a wide range of Ra_I. Following this procedure, the universal constants are determined to be $a = 0.051$ and $b = 0.87$. From equations (15) and (16) we have

$$\begin{cases} \dfrac{\varepsilon_H}{\alpha} = 0.051 Ra^{*0.87} \\ Ra^* = \dfrac{g\beta\Delta T^* L^3}{\alpha\nu}[\eta(1-\eta)]^{3.448} \end{cases} \quad (30)$$

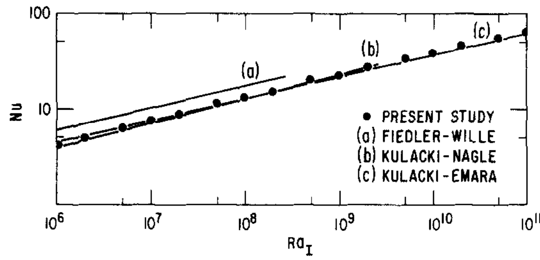

FIG. 2. Heat transfer from the upper surface of the fluid layer.

The mean Nusselt number is calculated based on equation (30) and shown graphically in Fig. 2. Also shown in the figure are the available empirical correlations for comparison. The calculated Nu–Ra relation can be represented by

$$Nu = 0.124 Ra^{0.25} \quad (31)$$

with an absolute error less than 7% over the range $1 \times 10^6 < Ra_I < 1 \times 10^{11}$. Using a least squares technique, the numerical data can be fitted equally well by a correlation of the form

$$Nu = 0.158 Ra^{0.237}. \quad (32)$$

Some calculated numerical values of Nu are given in Table 1. The theoretical prediction is found to be in good agreement with the experimental data over the entire range of Rayleigh numbers under consideration.

MECHANISM OF TURBULENT THERMAL CONVECTION

1. *Mean turbulent temperature fields*

The calculated mean temperature profiles for different Rayleigh numbers are shown in Fig. 3. Experimentally observed temperature distribution [8] at $Ra_I = 9.3 \times 10^7$ is also plotted in the figure for comparison. For $Ra_I \geqslant 1 \times 10^7$, the temperature profiles are found to be essentially flat except in the region near the upper surface. Because of the intensive turbulent mixing in the core, the stagnant fluid sublayers at the lower and upper surfaces are restricted to very thin regions. Since the lower wall is insulated, the temperature drop across the lower stagnant fluid sublayer is negligible. As a result, there is practically no thermal sublayer in the lower wall region, especially at high Rayleigh numbers. Temperature gradient is found to be important only in the region close to the upper wall. This

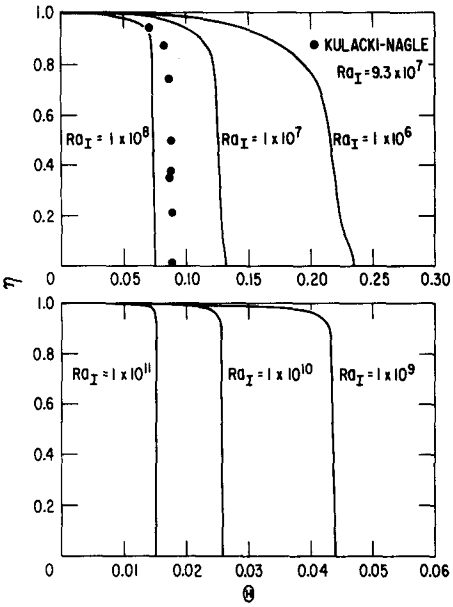

FIG. 3. Dimensionless temperature distribution in the fluid layer at various Rayleigh numbers.

result supports the conduction sublayer model according to which the total temperature drop across the fluid layer is considered to occur in the conduction sublayer at the upper surface. The thickness of the conduction sublayer is found to be of the same order of the dimensionless temperature at the lower boundary (Fig. 3), i.e. $\delta/L \sim \Theta_0$. From equation (23), the value of Θ_0 is the reciprocal of the mean Nusselt number. Thus, the thickness of the conduction sublayer varies according to $\delta/L \sim Nu^{-1}$. This dependence has been observed experimentally in both types of turbulent thermal convections with and without internal energy sources [8, 10, 17].

At $Ra_I \sim 1 \times 10^6$, the convection flow is still within the transition regime. Turbulent mixing in the center core is not so intensive compared to those at higher Rayleigh numbers. Consequently, eddy heat transport is less effective at this Rayleigh number, so that the mean temperature gradient is not negligible in the turbulent core. At the same time, temperature drop across the lower stagnant fluid sublayer becomes important. The thermal boundary sublayer at the lower surface, though much less remarkable than the one at the upper wall, is no longer trivial (Fig. 3). Thus the conduction sublayer model fails to apply at $Ra_I < 1 \times 10^6$.

2. *Eddy heat transport and production of thermal variance*

The rate of eddy heat transfer at different locations of the heated fluid layer can be related to the internal Rayleigh number and the local mean temperature by use of equations (2), (21), and (30). In dimensionless variables, the result is

$$\overline{W\Theta'} = \frac{0.102 Ra_I^{0.87} \Theta^{0.87} \eta^4 (1-\eta)^3}{\Theta_0[1+0.051 Ra_I^{0.87} \Theta^{0.87} \eta^3 (1-\eta)^3]}, \tag{33}$$

where $\overline{W\Theta'}$ is the dimensionless eddy heat transport defined by

$$\overline{W\Theta'} = \frac{\overline{vT'}}{\alpha(T_0 - T_1)/L}. \tag{34}$$

From considerations of the turbulent energy equation, the production of thermal variance is represented by the term $\overline{vT'}$ (dT/dz). In dimensionless variables, this term can be related to the internal Rayleigh number and the mean turbulent temperature field as

$$\overline{W\Theta'} \frac{d\Theta}{d\eta} = \frac{0.204 Ra_I^{0.87} \Theta^{0.87} \eta^5 (1-\eta)^3}{\Theta_0[1+0.051 Ra_I^{0.87} \Theta^{0.87} \eta^3 (1-\eta)^3]^2}. \tag{35}$$

The calculated eddy heat transport and production of thermal variance are presented in Figs. 4 and 5, respectively, with Ra_I as the parameter.

The eddy heat flux is found to increase almost linearly with distance from the lower wall and reaches a peak value at a location just outside the conduction sublayer at the upper boundary. This peak value is a function of the internal Rayleigh number and is found to be proportional to $Ra_I^{1/4}$. The region of linear eddy heat transport corresponds closely to the region of

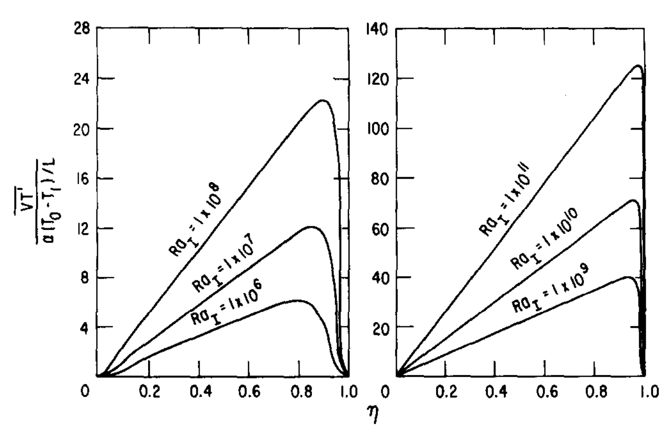

FIG. 4. Dimensionless eddy heat transport in turbulent thermal convection with uniform volumetric energy sources.

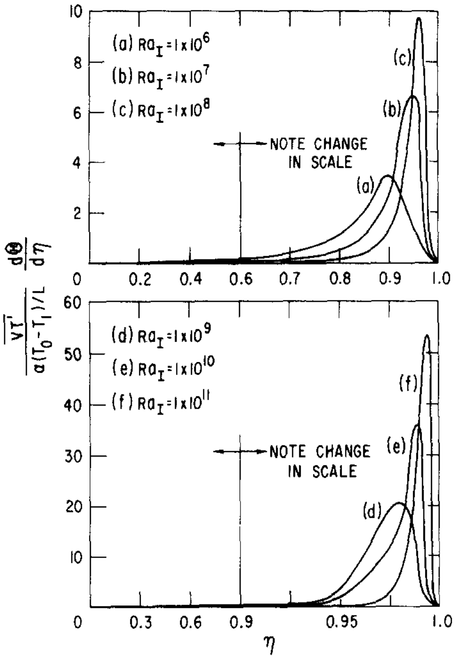

FIG. 5. Production of thermal variance in turbulent thermal convection with uniform volumetric energy sources.

constant temperature distribution. This indicates that turbulence is the predominant mode of heat transfer in those regions. For a volumetrically heated fluid layer, the local heat flux must be a linearly increasing function of distance from the lower wall. Therefore, the eddy heat flux distribution must also be linear in the turbulent core. As Ra_I increases, both the eddy heat flux and the thickness of the linear region increase. A similar result was observed experimentally by Kulacki and Goldstein [16] in a horizontal layer with equal surface temperatures. The linearity of the eddy heat transport in the turbulent mixing core seems to be independent of the boundary conditions at the upper and lower surfaces, provided that the internal Rayleigh number is high enough to maintain a strong turbulent mixing in the core.

As shown in Fig. 5, production of thermal variance is essentially zero in the lower 75–95% of the layer, depending upon the corresponding values of Ra_I. The production is found to be significant only near the upper wall. For all Ra_I, peak values are obtained in production in the region of the order of δ from the upper surface. This is quite expected since within the turbulent core the mean temperature gradient is largest there. The occurrence of the peak production corresponds approximately to the location of maximum eddy heat flux. Physically this result is in good agreement with the observation that the temperature fluctuation is very small in the lower portion of the layer and is greatest at a position near the upper wall [8].

During the heat transfer process, the turbulent thermal energy produced in the upper wall region is being diffused and convected gradually into the turbulent mixing core. At the same time, a portion of the turbulent energy is being dissipated by the mixing process. At steady state, the overall balance is such that the turbulent energy distribution be maintained constant in the mixing core. For larger Ra_I, the peak production is larger, resulting in higher turbulent energy level in the core. This increases the rate of eddy heat transport and thus the total rate of heat removal from the heated fluid layer.

3. *Effect of Prandtl number*

The eddy heat transport model described above does not include the effect of Prandtl number on turbulent thermal convection in a horizontal heated fluid layer. In general, the eddy thermal diffusivity given by equation (30) is considered to be valid only for moderate Prandtl numbers. For fluids with very large or very small Prandtl numbers, the thickness of the conduction sublayer may not correspond to that of the stagnant fluid sublayer. In particular, if Pr is very small, the conduction sublayer will be very thick compared to the stagnant fluid sublayer. If Pr is very large, on the other hand, there will be a thin conduction sublayer imbedded in a thick stagnant fluid sublayer. This physical argument has been used by Kraichnan [19] and Long [20] to estimate the effect of Prandtl number on turbulent Rayleigh–Benard convection. Depending on Pr, the conduction and stagnant fluid sublayers may not overlap each other. Therefore, turbulent mixing may or may not be important at the edge of the conduction sublayer. Hence, for extreme values of Pr, the ratio ε_H/α as given by equation (30) must be modified. One possible way of doing that is to use a modified local Rayleigh number, Ra^*_{modified}, in place of Ra^* in equation (30), where

$$Ra^*_{\text{modified}} = \frac{g\beta\Delta T^* l^{*3}}{\alpha^{1+n}\nu^{1-n}} = Ra^*\, Pr^n. \tag{36}$$

The constant n in equation (36), however, must be determined by experiments.

SUMMARY AND CONCLUSIONS

Natural convection in a volumetrically heated fluid layer at high Rayleigh numbers is treated analytically by introduction of the eddy heat transport model. The model enables the determination of the boundary heat fluxes, the mean turbulent temperature fields, the distributions of eddy heat transport and production of thermal variance. At high Rayleigh numbers, the mean temperature is found to be essentially constant throughout the layer except in a sublayer region near the upper wall. The thickness of such a region is found to be inversely proportional to the mean Nusselt number. Outside the sublayer region, the distribution of eddy heat flux is linear for all Rayleigh numbers under consideration. Production of thermal variance is negligible in the lower 75–95% of the layer and is greatest near the upper boundary. Comparison of the heat transfer predictions with measurements shows an excellent agreement within the turbulent thermal convection regime.

The present approach provides a simple method of determination of the average and the turbulent heat-transfer characteristics in a volumetrically heated fluid layer. So far, this has been the first attempt to treat heat transfer problems of this type without having to consider the stability of the flow. It is felt that, in terms of the local Rayleigh number, the eddy heat transport model described in this report can be generalized to horizontal fluid layers heated from below and to cases with combined internal and external Rayleigh number effects. Work is being done in this direction at Argonne. Results will be reported in the near future.

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CONVECTION NATURELLE DANS UNE COUCHE FLUIDE AVEC CHAUFFAGE VOLUMIQUE AUX NOMBRES DE RAYLEIGH ELEVES

Résumé—On a développé dans cette étude un modèle phénoménologique du transfert de chaleur turbulent en convection naturelle avec sources d'énergies volumiques, aux nombres de Rayleigh élevés. Le modèle est appliqué au problème de la convection thermique dans une couche horizontale de fluide chauffé avec une frontière inférieure adiabatique et une paroi supérieure isotherme. Une formule est obtenue donnant le nombre de Nusselt moyen en régime thermique établi. On présente les résultats relatifs aux distributions de température moyenne, de flux de chaleur turbulent et de production de variance thermique dans le fluide chauffé. On discute les mécanismes de la convection thermique turbulente aux nombres de Rayleigh élevés. La comparaison effectuée avec les expériences existantes a fourni un bon accord.

NATÜRLICHE KONVEKTION IN EINER MIT KONTINUIERLICH ÜBER DAS VOLUMEN VERTEILTEN WÄRMEQUELLEN VERSEHENEN FLUIDSCHICHT BEI HOHEN RAYLEIGH-ZAHLEN

Zusammenfassung—Es wird phänomenologisch ein Modell des turbulenten Wärmetransports bei natürlicher Konvektion mit kontinuierlich über das Volumen verteilten Wärmequellen bei hohen Rayleigh–Zahlen entwickelt. Das Modell wird angewandt auf das Problem der thermischen Konvektion in einer horizontalen, beheizten Fluidschicht mit adiabater Unterseite und isothermer Oberseite. Für den stationären Fall wird eine Beziehung für die mittlere Nusselt–Zahl aufgestellt. Die Verteilung der örtlichen, zeitlich gemittelten Temperatur des Wärmestromes und die Bildung thermischer Unterschiede im beheizten Fluid werden angegeben. Der Mechanismus der turbulenten natürlichen Konvektion bei hohen Rayleigh–Zahlen wird diskutiert. Der Vergleich mit vorhandenen Versuchsergebnissen zeigt gute Übereinstimmung.

СВОБОДНАЯ КОНВЕКЦИЯ В СЛОЕ ЖИДКОСТИ С ОБЪЕМНЫМИ ТЕПЛОВЫДЕЛИТЕЛЯМИ ПРИ БОЛЬШИХ ЧИСЛАХ РЕЛЕЯ

Аннотация — Разработана феноменологическая модель турбулентного свободноконвективного переноса тепла при больших числах Релея и наличии объемных источников энергии. С помощью данной модели решается задача тепловой конвекции в горизонтальном нагреваемом слое жидкости, ограниченном снизу адиабатической границей, а сверху — изотермической стенкой. Для средних значений числа Нуссельта получено корреляционное соотношение для стационарного процесса теплообмена. Представлены профили средней температуры турбулентного потока жидкости, турбулентного теплового потока и пульсации температуры в нагретой жидкости. Рассмотрен механизм турбулентной тепловой конвекции при больших числах Релея. Полученные результаты сравниваются с имеющимися экспериментальными данными и наблюдается хорошее соответствие между ними.